PhyzExamples: Advanced Waves

Physical Quantities • Symbols • Units • Brief Definitions

Wavelength • λ • meter: m • The distance through which a complete cycle of a wave is observed (e.g. from one crest to the next crest).
Amplitude • A or x_{max} • meter: m • The distance between the equilibrium position of a medium and the top of a crest or bottom of a trough of a wave passing through it. Indicative of the energy associated with the wave.
Frequency • f or ν (nu) • hertz: Hz • The rate at which a source or observer of waves oscillates. Related to the rate at which a source transmits energy to an observer.
Period • T • seconds: s • The time required for a source or observer of waves to oscillate through one cycle. Be careful: T also represents tension.
Speed • v_w • meters per second: m/s • The rate at which a wave passes or propagates through a medium or through space.

Equations

$v_w = f\lambda$ • The Wave Equation • *wave speed = frequency · wavelength*
$f_{obs} = f_s[(v_w - v_{obs})/(v_w - v_s)]$ • The Doppler Effect • *observed frequency = source frequency · the difference of observer speed from wave speed / the difference of source speed from wave speed*
$v_w = \sqrt{(TL/m)}$ • *speed of a wave in a stretched cord = square root of (cord tension · cord length / cord mass)*
$\lambda_n = 2L/n$ • Fixed Ends Resonance • *wavelength of the nth harmonic = two · the length of the oscillating medium / the harmonic number*
$f_n = nv_w/2L$ • Fixed Ends Resonance • *frequency of the nth harmonic = the harmonic number · wave speed / two · length of the oscillating medium*

Smooth Operations Examples

1. Ripples on a pond pass a rock at a frequency of 5 Hz; there is a distance of 3 cm between the wave crests. What is the speed of the waves?

1. $f = 5$ Hz $\lambda = 0.03$ m $v = ?$
$v = f\lambda$
$v = 5 \text{ Hz} \cdot 0.03 \text{ m}$
$\underline{v = 0.15 \text{ m/s}}$

2. Waves in a wave machine travel at 60cm/s. If one end of the wave sticks is wiggled once every two seconds, what wavelength will be produced?

2. $v = 0.60$ m/s $T = 2$ s $(f = 1/T = 0.5\text{hz})$ $\lambda = ?$
$v = f\lambda$ $\lambda = v/f$
$\lambda = 0.60 \text{ m/s} / 0.5 \text{ Hz}$
$\underline{\lambda = 1.2 \text{ m}}$

3. A train whistle blows at 750 Hz as it approaches a railroad crossing. If the train is moving at 25 m/s, what is the frequency heard by an observer standing near the crossing? The speed of sound in air at room temperature is 343 m/s.

3. $f_s = 750$ Hz $v_s = 25$ m/s $v_{obs} = 0$ $f_{obs} = ?$
$f_{obs} = f_s [(v_w - v_{obs}) / (v_w - v_s)]$
$f_{obs} = 750 \text{ Hz}[(343 \text{ m/s})/(343 \text{ m/s} - 25 \text{ m/s})]$
$\underline{f_{obs} = 810 \text{ Hz}}$

4. The 400 Hz horn of a stationary car is heard as 380 Hz by a car moving away from it. How fast is the moving car traveling?

4. $f_s = 400$ Hz $f_{obs} = 380$ Hz $v_s = 0$ $v_{obs} = ?$
$f_{obs} = f_s [(v_w - v_{obs}) / (v_w - v_s)]$
$f_{obs} / f_s = (v_w - v_{obs}) / (v_w)$
$f_{obs}v_w / f_s = v_w - v_{obs}$
$f_{obs}v_w / f_s - v_w = - v_{obs}$
$v_{obs} = v_w - (f_{obs}v_w / f_s)$
$v_{obs} = 343 \text{ m/s} - (380 \text{ Hz} \cdot 343 \text{ m/s} / 400 \text{ Hz})$
$\underline{v_{obs} = +17 \text{ m/s}}$ = 17 m/s away from horn

5. A wave travels 100 m/s in an 80 cm, 30 g cord. What is the tension in the cord?

5. $v = 100$ m/s $L = 0.8$ m $m = 0.03$ kg $T = ?$

$v = (TL/m)$

$T = v^2m/L$

$T = (100 \text{ m/s})^2 \cdot 0.03 \text{ kg} / 0.8 \text{ m}$

$\underline{T = 375 \text{ N}}$

6. A 25 g, 1.2 m string is under 200 N of tension. Determine the wavelength and frequency of the fundamental.

6. $m = 0.025$ kg $L = 1.2$ m $T = 200$ N

$\lambda = ?$ $f = ?$ $n = 1$

$\lambda = 2L/n$

$\lambda = 2 \cdot 1.2 \text{ m} / 1 = \underline{2.4 \text{ m} = \lambda}$

$f = v_w \cdot n/2L = (TL/m) \cdot n/2L = n/2 \ (T/mL)$

$f = 1/2 \cdot (200 \text{ N} / 0.025 \text{ kg} \cdot 1.2 \text{ m}) = \underline{41 \text{ Hz} = f}$

Welcome to the Real World Examples

7. A cord is anchored to a wall at one end. It passes over a pulley and is attached to a mass at the other end. When the cord is plucked, it vibrates in its fundamental mode. The sound waves that emerge from the cord have a wavelength of 78cm. The cord is 63cm long and has a mass of 5.4g.

a. What is the resonant frequency of the cord?

$v_W = f\lambda$ $f = v_W/\lambda$

$f = 343 \text{ m/s} / 0.78 \text{ m} \underline{= 440 \text{ Hz}}$ (frequency of the sound waves = frequency of the waves in the cord)

b. What is the wavelength of the fundamental mode of vibration in the cord?

$\lambda = 2L/n$

$\lambda = 2(0.63 \text{ m}) / 1 \underline{= 1.26 \text{ m}}$

c. What is the speed of a wave in the cord?

$v_w = f\lambda$

$v_w = 440 \text{ Hz} \cdot 1.26 \text{ m} \underline{= 554 \text{ m/s}}$

d. What is the weight of the mass?

$v_w = (TL/m)$ $T = v_w{}^2m/L$

$T = (554 \text{ m/s})^2 \cdot 0.0054 \text{ kg} / 0.63 \text{ m} \ \underline{= 2630 \text{ N}}$

8. A radar gun sends out waves with a frequency *A* toward a moving object. The waves reflected from the moving object return to the radar gun with a frequency *B*. The waves travel with a speed *r*; the radar gun is at rest. What is the speed *v* of the moving object? If *A*=100,000.000 Hz, *B*=100,000.020 Hz, and *r*=300,000,000 m/s, what is *v*?

a. Determine the frequency (C) at which the waves arrive at the moving object. The object (observer in this case) moves with a speed +v; the radar gun (source) is at rest, the waves move with speed +r.

$C = A [(r - v) / r]$

b. The waves are thus emitted from the car (back to the radar gun) at frequency C. Now determine an expression for the frequency B at which the waves return to the radar gun. The source is now the object which moves with a speed –v (opposite the direction from source to observer).

$B = C [(r / (r + v)]$

$B = A [(r - v) / r] \cdot [r / (r + v)]$

$B (r + v) = A (r - v)$

$Br + Bv = Ar - Av$

$Bv + Av = Ar - Br$

$v = r (A - B)/(A + B)$

$v = 300{,}000{,}000 \text{ m/s} (100{,}000.000 \text{ Hz} - 100{,}000.020 \text{ Hz})/(100{,}000.000 \text{ Hz} - 100{,}000.020 \text{ Hz})$

$\underline{v = -30 \text{ m/s}}$ (object is moving at 30 m/s toward radar gun)