PhyzExamples: Simple Harmonic Motion

Physical Quantities • Symbols • Units • Brief Definitions

Restoring Force • *F* • newton: N • Force acting on a body displaced from a position of stable equilibrium. It acts in a direction so as to return (restore) the body to equilibrium.

Force Constant or **Spring Constant** • *k* • newton per meter: N/m • A measure of the stiffness of an elastic object, typically a spring. The quantity of force required to stretch the object by a particular distance.

Elastic Potential Energy • *PE* • joule: J • Energy stored in a system when a body is displaced from its position of stable equilibrium. For example, a stretched or compressed spring, a stretched rubber band, a stretched archer's bow.

Period • *T* • seconds: s • The time required for one cycle of a periodic motion.

Equations

$F = kx$ • Hooke's Law • *restoring force = force constant · distance from equilibrium.*

$PE = {}^1/_2 kx^2$ • *Elastic Potential Energy = one half · force constant · distance from equilibrium squared.*

$T = 2\pi\sqrt{(-x/a)}$ • General SHM • *period = two · pi · square root of negative the distance from equilibrium / acceleration.*

$T = 2\pi\sqrt{(m/k)}$ • Spring-Mass Oscillator • *period = two · pi · square root of the oscillating mass / force constant.*

$T = 2\pi\sqrt{(L/g)}$ • Simple Pendulum • *period = two · pi · square root of the length of the pendulum / gravitational acceleration in the region.*

Smooth Operations Examples

1. What is the force constant of a spring that is compressed 25 mm under a load of 1800 N?

1. x = 0.025 m   F = 1800 N   k = ?
F = kx
k = F/x
k = 1800 N / 0.025 m
<u>k= 72,000 N/m</u>

2. How far could a 72 kN/m spring be stretched if 36 kJ of work were done to stretch it?

2. k = 72 x 10^3 N/m   PE = W = 36 x 10^3 J
PE = (1/2)kx^2
x =   (2PE/k)
x =   (2 · 36 x 10^3 J / 72 x 10^3 N/m)
<u>R = 1 m</u>

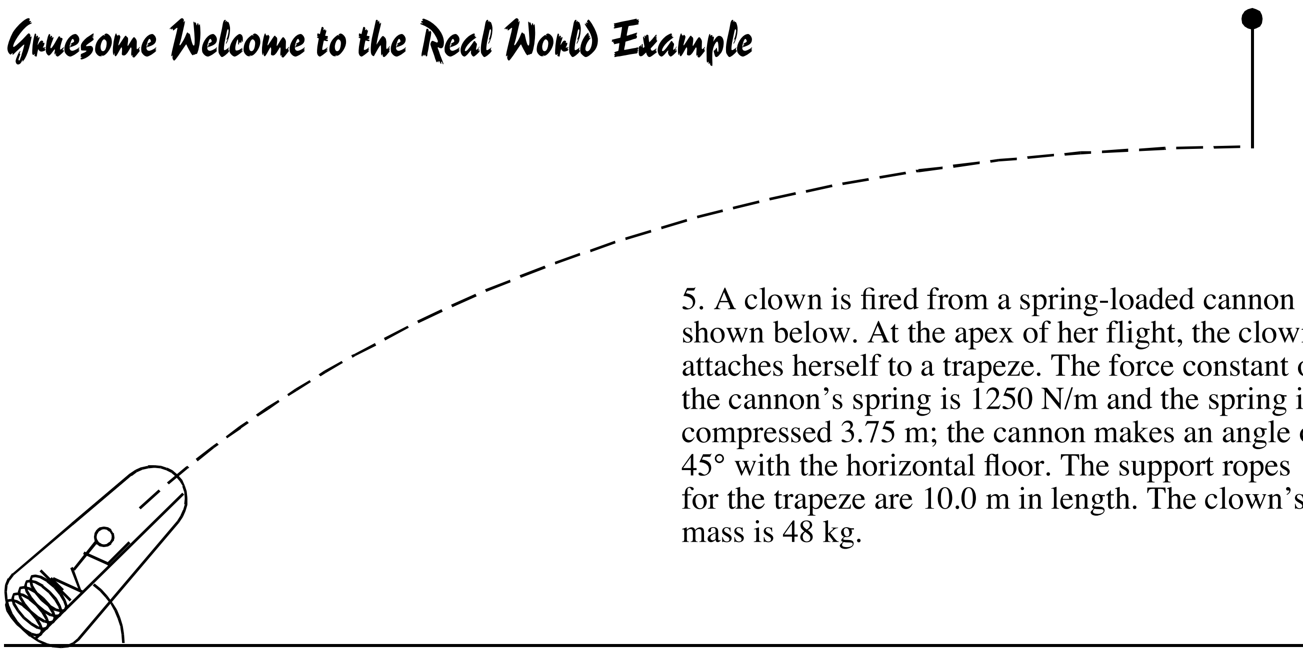
3. What is the mass of an object that oscillates 10 times in 42 s at the end of a 56 N/m spring?

3. T = 42 s / 10 = 4.2 s   k = 56 N/m   m=?
T = 2   (m/k) T^2 = 4 ^{2}m/k
m = T^2k/4 ^{2}
m = $(4.2\ s)^2$ · 56 N/m / 4 ^{2}
<u>m= 25 kg</u>

4. What is the acceleration of gravity on a planet where a 0.62 m pendulum has a period of 1.3 s?

4. T = 1.3 s   L = 0.62 m   g = ?
T = 2   (L/g) T^2 = 4 ^{2}L/g
g = 4 ^{2}L/T^2
g = 4 ^{2} · 0.62 m / $(1.3\ s)^2$
<u>g= 14 m/s^2</u>

Gruesome Welcome to the Real World Example



5. A clown is fired from a spring-loaded cannon as shown below. At the apex of her flight, the clown attaches herself to a trapeze. The force constant of the cannon's spring is 1250 N/m and the spring is compressed 3.75 m; the cannon makes an angle of 45° with the horizontal floor. The support ropes for the trapeze are 10.0 m in length. The clown's mass is 48 kg.

a. At what horizontal distance from the launch point does the clown reach the apex of her flight?

$d = R/2$

$d = v_0^2/2g$

$KE_{launch} = PE_{stored}$

$(1/2)mv_0^2 = (1/2)kx^2$

$v_0^2 = kx^2/m$

$d = kx^2/2mg = 1250\text{ N/m }(3.75\text{ m})^2 / 2(48\text{ kg})(9.8\text{ m/s}^2)$

$\underline{d = 18.7\text{ m}}$

b. How high above the floor is the apex of the flight?

$y=?\quad v_{y0} = v_0\sin\theta = x\sin\theta\ (k/m) = 3.75\text{ m}\cdot\sin 45°\ (1250\text{ N/m} / 48\text{ kg}) = 13.5\text{ m/s}\quad v_y = 0$

$a = -9.8\text{m/s}^2\quad t=?$

$v_y^2 = v_{y0}^2 + 2ay$

$y = -v_{y0}^2/2a = -(13.5\text{ m/s})^2 / 2(-9.8\text{ m/s}^2)$

$\underline{y = 9.34\text{ m}}$

c. If the trapeze bar has a mass of 10 kg, how high above the equilibrium position will the bar rise when the clown attaches herself to it?

$PE_{high} = KE_{low}$

$mgh = (1/2)mv_x'^2$

Find v_x' using cons of mom (inelastic collision) $\quad p' = p\quad mv_x' + m_tv_x' = mv_x\quad (m+m_t)v_x' = mv_x$

$v_x' = mv_x / (m + m_t)$

$v_x = v_0\cos\theta = x\cos\theta\ (k/m)$

$v_x' = mx\cos\theta\ (k/m) / (m + m_t) = 48\text{ kg }(3.75\text{ m})(\cos 45°)\ (1250\text{ N/m} / 48\text{ kg}) / (48\text{ kg} + 10\text{ kg})$

$v_x' = 11\text{ m/s}$

$h = v_x'^2 / 2g = (11\text{ m/s})^2 / 2(9.8\text{ m/s}^2)$

$\underline{h = 6.4\text{ m}}$

d. With what period will the clown swing from her perch?

$T = 2\quad (L/g) = 2\quad (10.0\text{ m} / 9.8\text{ m/s}^2)$

$\underline{T = 6.3\text{ s}}$