PhyzExamples: Rotation

Smooth Operations: Kinematics

1. [Rot UM] How long does it take for a record spinning at 33.3 rpm to rotate through 100 radians?

1. $\omega = 33.3$ rev/min x 2π rad/rev x 1 min/60s
$\omega = 3.49$ rad/s $\theta = 100$ rad $t = ?$
$\omega = \theta/t$
$t = \theta/\omega$
$t = 100$ rad / 3.49 rad/s
$\underline{t = 28.7\ s}$

2. [Rot UAM] What is the angular acceleration of a compact disc that begins at rest and accelerates to 50 rad/s in 1.5 s?

2. $\theta = ?$ $\omega_o = 0$ $\omega = 50$rad/s $\alpha = ?$ $t = 1.5$s
$\omega = \omega_o + \alpha t$
$\alpha = \omega/t$
$\alpha = 50$ rad/s / 1.5 s
$\underline{\alpha = 33.3\ rad/s^2}$

3. [Rot UAM] A tire on my PhyzVan had an angular acceleration of –5 rad/s^2. If the wheel was originally turning at 80 rad/s and turned through 500 rad during the deceleration, what was the final speed of the wheel?

3. $\theta = 500$ rad $\omega_o = 80$ rad/s $\omega = ?$
$\alpha = -5$ rad/s^2 $t = ?$
$\omega^2 = \omega_o^2 + 2\alpha\theta$
$\omega = \sqrt{(\omega_o^2 + 2\alpha\theta)}$
$\omega = \sqrt{((80\ rad/s)^2 + 2(-5\ rad/s^2)(500\ rad))}$
$\underline{\omega = 37\ rad/s}$

4. [Rolling] An oil barrel (r = 40 cm) rolls on a level surface at 6 m/s. What is the barrel's angular speed?

4. $v = 6$m/s $r = 0.4$ m $\omega = ?$
$v = r\omega$
$\omega = v/r$
$\omega = 6$ m/s / 0.4 m
$\underline{\omega = 15\ rad/s}$

Smooth Operations: Dynamics 1

5. How far must a force of 50 N be placed along a wrench so that a torque of 10 N·m can be achieved?

5. $F = 50$ N $\tau = 10$ N·m $r = ?$
$\tau = r \times F$
$r = \tau/F$
$r = 10$ N·m / 50 N
$\underline{r = 0.2\ m = 20\ cm}$

6. What is the mass of a basketball whose diameter is 30 cm and whose moment of inertia is 0.0075 kg·m^2?

6. $R = D/2 = 0.15$ m $I = 0.0075$ kg·m^2 $M = ?$
$I = (2/3)\ MR^2$ [hollow sphere]
$M = (3/2)\ I / R^2$
$M = (3/2)\ 0.0075$ kg·m^2 / $(0.15\ m)^2$
$\underline{M = 0.5\ kg}$

7. How much torque is needed to angularly accelerate a 3-kg·m^2 fan blade at 12 rad/s^2?

7. $I = 3$ kg·m^2 $\alpha = 12$ rad/s^2
$\tau = I\alpha$
$\tau = 3$ kg·m^2 · 12 rad/s^2
$\underline{\tau = 36\ N{\cdot}m}$ [The N·m here are NOT joules!]

8. What is the combined rotational inertia of Jenny and the rotating stool she's sitting on if a torque of 20 N·m causes an angular acceleration of 2 rad/s^2?

8. $\tau = 20$ N·m $\alpha = 2$ rad/s^2
$\tau = I\alpha$
$I = \tau/\alpha$
$I = 20$ N·m / 2 rad/s^2
$\underline{I = 10\ kg{\cdot}m^2}$

Smooth Operations: Dynamics 2 & 3

9. What is the angular momentum of a 20-g, 11.8-cm compact disc spinning at 500 rpm?

9. $m = 20\ g = 0.020\ kg$
$r = d/2 = 5.9\ cm = 0.059\ m$
$\omega = 500\ rev/min \times 2\pi\ rad/rev \times 1\ min/60\ s$
$\omega = 52.4\ rad/s$
$L = I\omega = (1/2)mr^2 \cdot \omega$
$L = (1/2)\ 0.020\ kg \cdot (0.059\ m)^2 \cdot 52.4\ rad/s$
$\underline{L = 0.0018\ kg{\cdot}m^2/s}$

10. How much work does Jearl do on the merry-go-round if he applies a torque of 120 N·m while accelerating it through 2π rad?

10. $\tau = 120\ N{\cdot}m \quad \theta = 2\pi\ rad$
$W = \tau\theta$
$W = 120\ N{\cdot}m \cdot 2\pi\ rad$
$\underline{W = 754\ J}$ [Yes, the radians disappear and the N·m become joules. Weird, but true!]

11. To what angular speed did Jearl accelerate the merry-go-round if its rotational inertia was 300 kg·m²?

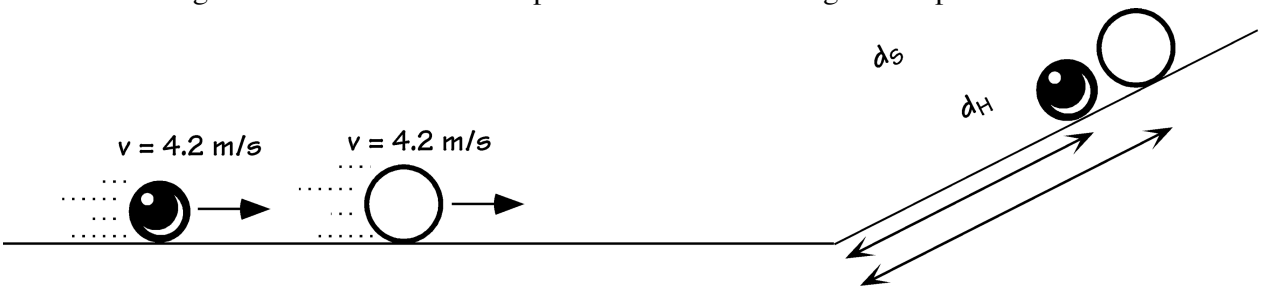
11. $KE = 754\ J \quad I = 300\ kg{\cdot}m^2$
$KE = (1/2)I\omega^2$
$\omega = \sqrt{(2KE / I)}$
$\omega = \sqrt{(2 \cdot 754\ N{\cdot}m / 300\ kg{\cdot}m^2)}$
$\underline{\omega = 2.24\ rad/s}$

12. What is the angular speed of a bit on a 500 W electric drill that provides 6.25 N·m of torque?

12. $P = 500\ W \quad t = 6.25\ N{\cdot}m$
$P = \tau{\cdot}\omega$
$\omega = P/\tau$
$\omega = 500\ W / 6.25\ N{\cdot}m$
$\underline{\omega = 80\ rad/s}$

Welcome to the Real World Example

13. A hoop (hollow cylinder) and a solid sphere are rolling along a level surface at 4.2 m/s when they encounter an incline of 27°. *Notice that I'm not saying what the mass or radius of either object is!*
a. How far along the incline will the hoop roll before coming to a stop?
b. How far along the incline will the solid sphere roll before coming to a stop?



13a. $v = 4.2\ m/s \quad \theta = 27° \quad I = mr^2$ (hoop)
height above level surface
$KE_{BOT} = PE_{TOP}$
$KE_{LIN} + KE_{ROT} = PE_{TOP}$
$(1/2)mv^2 + (1/2)I\omega^2 = mgh$
$(1/2)mv^2 + (1/2)(mr^2)(v/r)^2 = mgh$
$(1/2)mv^2 + (1/2)(mv^2) = mgh$
$v^2 = gh$
$h = v^2/g$
distance along incline
$d_H = h/\sin\theta$
$d_H = v^2/g\sin\theta$
$d_H = (4.2\ m/s)^2 / 9.8\ m/s^2\ \sin 27°$
$\underline{d_H = 3.97\ m}$

13b. $v = 4.2\ m/s \quad \theta = 27° \quad I = (2/5)mr^2$ (SS)
height above level surface
$KE_{BOT} = PE_{TOP}$
$KE_{LIN} + KE_{ROT} = PE_{TOP}$
$(1/2)mv^2 + (1/2)I\omega^2 = mgh$
$(1/2)mv^2 + (1/2)(2/5)(mr^2)(v/r)^2 = mgh$
$(1/2)mv^2 + (2/10)(mv^2) = mgh$
$(7/10)v^2 = gh$
$h = 7v^2/10g$
distance along incline
$d_S = h/\sin\theta$
$d_S = 7v^2/10g\sin\theta$
$d_S = 7(4.2\ m/s)^2 / 10{\cdot}9.8\ m/s^2\ \sin 27°$
$\underline{d_S = 2.78\ m}$

Did you expect the same answer as for the hoop?