PhyzExamples: Advanced Electrostatics

Physical Quantities • Symbols • Units • Brief Definitions

Charge • *q or Q* • coulomb [*KOO lom*]: C • A characteristic of certain fundamental particles.
Elementary Charge • $e = 1.6 \times 10^{-19}$ C • The *quantity* of charge carried by protons and electrons.
Electric Field • *E* • newton per coulomb: N/C or volt per meter: V/m • The electric force experienced by each unit of charge in a particular location. Typically, the unit of charge is the coulomb.
Electric Potential Energy • *PE* • joule: J • Energy of position arising from electric forces acting on electric charge.
Electric Potential • *V* • joule per coulomb: J/C or volt: V • The electric potential energy held by each unit of charge in a particular location. Typically, the unit of charge is the coulomb.
Capacitance • *C* • coulomb per volt: C/V or farad: F • The quantity of charge held by either of two parallel plates for each unit of electric potential difference between the two plates.
Coulomb Constant • $k = 9 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}$.
Permittivity of Free Space • $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$
Masses • Electron: 9.11×10^{-31} kg • Proton: 1.67×10^{-27} kg • Neutron: 1.67×10^{-27} kg

Equations

$E = F/q$ • *electric field = electric force / charge*
$E = kQ/R^2$ • *electric field near a point or spherical charge =coulomb constant · charge on point or sphere / square of distance from point or center of sphere*
$E = 4\pi kQ/A$ • *uniform electric field between plates = 4 · π · coulomb constant · plate charge / plate area*
$PE = qEd$ •*electric potential energy = charge of body in an electric field · electric field · distance that charge is moved through electric field*
$V = PE/q$ • *electric potential = electric potential energy / charge*
$V = kQ/R$ • *electric potential near a spherical charge = coulomb constant · charge / distance from spherical charge*
$V = Ed$ • *electric potential = electric field · distance between endpoints of the field*
$C = Q/V$ • *capacitance = the charge on either of two plates / electric potential difference between the plates*
$C = \varepsilon_0 A/d$ • *capacitance = the permittivity of free space · area of one plate / distance between the plates*
$PE = {}^1/{}_2QV$ • *energy stored in a capacitor =half the charge separated · potential across the plates*
$PE = {}^1/{}_2CV^2$ • *energy stored in a capacitor =half the capacitance · square of the potential across the plates*
$PE = Q^2/2C$• *energy stored in a capacitor = square of the charge separated / two times the capacitance*

Smooth Operations Examples

1. At a distance of 7.4 m from the center of a spherical charge there is an electric field of 25 kN/C. What is the charge on the sphere?

1. $R = 7.4$ m $E = 25{,}000$ N/C $Q = ?$
$E = kQ/R^2$
$Q = ER^2/k$
$Q = (25{,}000\ \mathrm{N/C})(7.4\ \mathrm{m})^2 / 9 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}$
<u>$Q = 0.000152$ C = 152 μC</u>

2. What is the radius of each of two circular plates with opposite charges of 200 nC with a 100,000 V/m electric field between them?

2. $Q = 200 \times 10^{-9}$ C $E = 100{,}000$ V/m
E=4 kQ/A   A= r^2 E=4 kQ/ r^2
r= (4kQ/E)
r = [(4)($9 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}$)($200 \times 10^{-9}$ C) / 100,000 V/m]
<u>r = 0.27 m</u>

3. What is the strength of a uniform electric field in which 1.3 J of energy are required to move an object with 8.6 mC of charge a distance of 34 mm?

3. PE = W = 1.3 J $q = 8.6 \times 10^{-3}$ C
   d = 0.034 m
PE=qEd
E=PE/qd
E = 1.3 J / 8.6×10^{-3} C · 0.034 m
E = 4400 J/C·m = 4400 N/C

4. What is the electric potential at a point in space in which an object with a 2.5 μC charge has 4.7 J of potential energy?

4. $q = 2.5 \times 10^{-6}$ C   PE = 4.7 J   V = ?
V = PE/q
V = 4.7 J / 2.5×10^{-6} C
V = 1,900,00 V = 1.9 MV

5. How far from the center of a spherical charge of 1.0 C would the electric potential be 9.0 V?

5. Q = 1.0 C   V = 9.0 V   R = ?
V = kQ/R
R = kQ/V
R = 9×10^{9} N·m^2/C^2 · 1.0 C / 9.0 V
R = 1.0×10^{9} m (> 620,000 mi)

6. Two parallel charged plates have a potential of 7500 V and a uniform electric field of 125,000 V/m between them. How far are the plates from each other?

6. V = 7500 V   E = 125,000 V/m
V = Ed
d = V/E
d = 7500 V / 125,000 V/m
d = 0.06 m = 6 cm

7. What is the capacitance of two parallel plates if each holds a charge of 12 mC when there is an electric potential difference of 3.0 V between them?

7. Q = 12×10^{-3} C   V = 3.0 V   C = ?
C = Q/V
C = 12×10^{-3} C / 3.0 V
C = 4.0×10^{-3} C/V = 4.0 mF

8. How much charge can be held on each plate of a 25μF capacitor charged to 120V?

8. C=25×10^{-6}C   V=120V   Q=?
C = Q/V
Q = CV
Q = 25×10^{-6} F · 120 V
Q = 0.003 FV = 3.0 mC

9. What is the separation between two parallel plates if they each have an area of 0.47 m^2 and have a capacitance of 7300 pF?

9. A = 0.47 m^2 C = 7300×10^{-12} F   d = ?
C = ε_0A/d
d = ε_0A/C
d = 8.85×10^{-12}C^2/N·m2 · 0.47m^2 / 7300×10^{-12}F
d = 0.00057m = 0.57mm

10. How much energy is stored in a capacitor if it holds 3.6 C of charge separated at a potential of 1.4 V?

10. Q = 3.6 C   V = 1.4 V   PE = ?
PE = QV/2
PE = 3.6 C · 1.4 V / 2
PE = 2.5 J

11. What is the capacitance of a capacitor that stores 70 mJ of energy when charged with a 6.0 V battery?

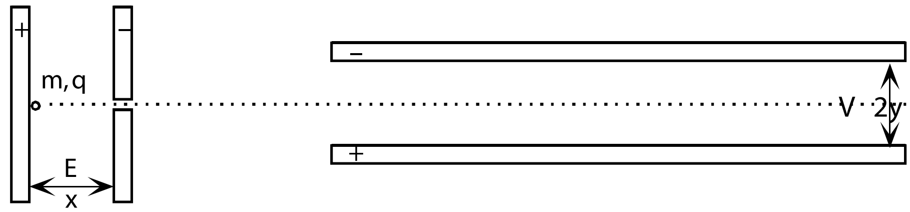
11. PE = 70×10^{-3} V = 6.0 V   C = ?
PE = CV^2/2
C = 2PE/V^2
C = 2(70×10^{-3} J) / (6.0 V)^2
C = 0.0039 F = 3900 μF

12. How much charge is separated in a 1200 μF capacitor if it stores 6.4 J of energy?

12. C = 1200×10^{-6} F   6.4 J
PE = Q^2/2C
Q =   (2C·PE)
Q =   (2 · 1200×10^{-6} F · 6.4 J)
Q = 0.12 C

Nasty Welcome to the Real World Example

9. A particle of mass m and positive charge q is accelerated through a uniform field E created by parallel plates a distance x apart. The particle passes through a hole in the negatively charged plate. Later, it enters a vertical field created by two oppositely charged plates separated by a distance $2y$ and charged to a potential difference of V as shown in the diagram.



a. Indicate the directions of the horizontal and vertical electric fields and sketch the path of the particle.

b. To what speed is the particle accelerated in the horizontal field?

$KE_{gained} = PE_{lost}$

$^1/_2 mv^2 = qEx$

$\underline{v = (2qEx/m)}$

c. What will be the magnitude of the vertical acceleration of the particle in the vertical field?

$F = ma$

$a = F/m \quad F = qE \quad E = V/d = V/2y$

$\underline{a = qV/2ym}$

d. How far will the particle travel in the vertical field before striking a plate?

2D kinematics

$x = d = ?$	$y = y$	find t	find d
$v_x = (2qEx / m)$	$v_{y0} = 0$	$y = v_{y0}t + 1/2at^2$	$d = v_x t$
	$v_y = ?$	$y = {}^1/_2 at^2$	$d = (2qEx/m) \cdot (4y^2m/qV)$
	$a = qV/2ym$	$t = (2y/a)$	$\underline{d = 2y\ (2Ex/V)}$
$t = ?$		$t = (2y \cdot 2ym/qV)$	
		$t = (4y^2m/qV)$	